

# Extraction of $\alpha$ From the CP Asymmetry in $B^0/\bar{B}^0 \rightarrow \pi^+\pi^-$ Decays

G. Kramer<sup>a</sup>, W. F. Palmer<sup>b</sup> and Y. L. Wu<sup>b</sup>

<sup>a</sup>II. Institut für Theoretische Physik<sup>1</sup>  
der Universität Hamburg,  
D-22761 Hamburg, Germany

<sup>b</sup>Department of Physics, The Ohio State University <sup>2</sup>,  
Columbus, Ohio 43210, USA

## Abstract

The influence of strong and electroweak penguin amplitudes in  $B/\bar{B} \rightarrow \pi^+\pi^-$  is investigated in connection with the determination of the unitarity triangle angle  $\alpha$  of the CKM matrix. A relation between the observable asymmetry, the angle  $\alpha$ , and the penguin amplitude is established. A model calculation of the penguin amplitude shows that the CP asymmetry in  $B^0 \rightarrow \pi^+\pi^-$  decays is only mildly influenced by the penguin amplitudes. Experimental limits on pure penguin and penguin dominated processes are consistent with the model. This information also suggests in a rather model independent way that penguin amplitudes will not be a serious complicating factor in the determination of  $\alpha$  from the  $\pi^+\pi^-$  time dependent asymmetry.

---

<sup>1</sup>Supported by Bundesministerium für Forschung und Technologie, 05 6 HH 93P(5), Bonn, Germany and EEC Program “Human Capital and Mobility” Network “Physics at High Energy Colliders” CHRX-CT93-0357 (DG 12 COMA)

<sup>2</sup>Supported in part by the US Department of Energy under contract DOE/ER/01545-605.

# 1 Introduction

It is expected that B decays will show large CP- violating effects, characterized by non-vanishing values of the angles  $\alpha$ ,  $\beta$  and  $\gamma$  in the unitarity triangle [1]. One of the best ways to detect this CP violation is to measure an asymmetry between  $B^0$  and  $\bar{B}^0$  decays into a CP eigenstate. If only one weak amplitude contributes to the decay, the phase in the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix can be extracted without uncertainties due to unknown hadronic matrix elements. Thus  $\sin 2\alpha$ ,  $\sin 2\beta$  and  $\sin 2\gamma$  can in principle be measured in  $B^0$ ,  $\bar{B}^0 \rightarrow \pi^+\pi^-$ ,  $J/\psi K_S$  and  $B_s$ ,  $\bar{B}_s \rightarrow \rho^0 K_S$  decays, respectively. Unfortunately the situation is more complicated. In all of the above cases, in addition to the tree contribution there are amplitudes due to strong and electroweak penguin diagrams. In the case of the  $J/\psi K_S$  final state the weak phase of the penguin term is the same as that of the tree contribution. Thus there is no uncertainty for determining  $\sin 2\beta$  from the CP asymmetry.

For  $B^0$ ,  $\bar{B}^0 \rightarrow \pi^+\pi^-$  the weak phases of the tree and penguin contributions are different causing hadronic uncertainties in the interpretation of an otherwise clean experiment. However, by measuring also the rates of  $B^0 \rightarrow \pi^0\pi^0$ ,  $B^+ \rightarrow \pi^+\pi^0$  and their charge conjugate decays one can isolate the amplitudes contributing to final states with isospin 0 and 2 and thereby determine  $\alpha$  [2]. This construction, however, relies on the fact that electroweak penguin contributions do not exist, since they contribute to both isospins and not only to  $I = 0$  as the strong penguin terms [3]. Although these weak penguin terms are expected to be small compared to the tree amplitudes [4], so that the Gronau-London construction should be possible, there is still the problem that the partial rates of the decays  $B^0$ ,  $\bar{B}^0 \rightarrow \pi^0\pi^0$  are at least an order of magnitude smaller than for the other  $2\pi$  final states [5]. In addition, because of two neutral pions in the final state, these decays are very difficult to measure accurately. So if this program can not be carried out the error of  $\sin 2\alpha$  is of the order of  $|P/T|$ , where  $P$  ( $T$ ) represents the penguin (tree) contribution to  $B^0 \rightarrow \pi^+\pi^-$ . In this connection DeJongh and Sphicas studied the behavior of the asymmetry based on a general parameterization of the penguin magnitude and phase [6].

Recently, two of us [5] calculated the effect of strong and electroweak penguins in all  $B^{\pm,0} \rightarrow \pi\pi$ ,  $\pi K$  and  $KK$  decays using specific dynamical models for the hadronic matrix elements. Concerning the asymmetry between  $\Gamma(B^0 \rightarrow \pi^+\pi^-)$  and  $\Gamma(\bar{B}^0 \rightarrow \pi^+\pi^-)$  ( $A_{CP}$ ) it turned out that the effect of electroweak penguins was indeed small, of the order of 2%, and that strong penguin amplitudes changed the asymmetry by less than 20% as compared to the tree value. These results were fairly independent of the specific models employed for calculating the hadronic matrix elements. Since the parameters in the time dependent asymmetry are obtained from *ratios* of the weak transition matrix elements, it is clear that they are much less model dependent than, for example, the branching ratio. Of course, this rather moderate change of  $A_{CP}$  for  $B^0 \rightarrow \pi^+\pi^-$  depends on  $P/T$  which was determined by the model calculations. Due to the way the results in [5] were presented only one particular set of CKM parameter values, namely  $\rho = -0.12$ ,  $\eta = 0.34$  was assumed. Although this is the preferred value obtained in the analysis of [7] in their so called “combined fit” it is certainly not the only possible set following from their analysis. From CP violation in the  $K^0 - \bar{K}^0$  system it is known that  $\eta \neq 0$ . Nevertheless, for both  $\rho$  and  $\eta$ , only very loose bounds exist which translate into similar loose bounds on the triangle phases  $\alpha$ ,  $\beta$  and  $\gamma$  [7]

$$10^\circ < \alpha < 150^\circ, \quad 5^\circ < \beta < 45^\circ, \quad 20^\circ < \gamma < 165^\circ. \quad (1)$$

From some predictive SUSY GUT models on fermion masses and mixings,  $\alpha$  was found to be large [8, 9]. For example, the model proposed in [9] predicted:  $\alpha = 86^\circ$ ,  $\beta = 22^\circ$ ,  $\gamma = 72^\circ$ .

It is clear that the change in  $A_{CP}$  for  $B^0/\bar{B}^0 \rightarrow \pi^+\pi^-$  due to penguin contributions depends not only on  $P/T$  but depends also on the particular set chosen for  $\rho$  and  $\eta$ . However, assuming fixed  $\rho$  and  $\eta$  values in [5] was an unnecessary limitation. Of course, it would be easy to repeat the calculation of [5] for any other set of  $\rho$ ,  $\eta$  inside the bounds of (1). This would give us a large array of numbers for  $A_{CP}$ . Instead we follow in this note a different route, which is particularly simple when we neglect the electroweak penguin terms and use some approximation for the strong penguins. We express the main contribution to the asymmetry parameter,  $a_{\epsilon+\epsilon'}$ , which is the coefficient of the  $\sin(\Delta mt)$  term in  $A_{CP}$  (see below) in terms of the tree and penguin amplitudes and their relative phase. Then  $a_{\epsilon+\epsilon'}$  depends only on  $\alpha$ . This gives us a clear insight into the dependence on  $|P/T|$  and on the strong phase and allows us to derive upper limits on the change of  $a_{\epsilon+\epsilon'}$  by including information from other decay channels which depend on the penguin contributions more strongly than the decay into  $\pi^+\pi^-$ .

The outline of the other sections is as follows. In section 2 we give the formulas of the asymmetry, from which we start and derive the formula for the change due to the penguin terms. In section 3 we present our results and discuss their relevance.

## 2 CP-violating Observables in $B^0 \rightarrow \pi^+\pi^-$

In this section we establish a relation between the CP-violating observables in  $B^0/\bar{B}^0 \rightarrow \pi^+\pi^-$ , the angle  $\alpha$  of the CKM matrix, and an auxiliary variable  $\alpha_0$  involving the ratio of penguin to tree amplitude and the strong interaction phase difference between the tree and the penguin amplitude. Applying the general analysis on rephase-invariant CP-violating observables given in ref. [10] for the B-system, and expressing the two physical mass eigenstates  $B_L$  and  $B_H$  as

$$B_L = p|B^0\rangle + q|\bar{B}^0\rangle, \quad B_H = p|B^0\rangle - q|\bar{B}^0\rangle \quad (2)$$

with the decay amplitudes of  $B^0 \rightarrow \pi^+\pi^-$  and  $\bar{B}^0 \rightarrow \pi^+\pi^-$  written as

$$g \equiv \langle \pi^+\pi^- | H_{eff} | B^0 \rangle = A_T e^{i\phi_T + i\delta_T} + A_P e^{i\phi_P + i\delta_P} \equiv \bar{h}, \quad (3)$$

$$h \equiv \langle \pi^+\pi^- | H_{eff} | \bar{B}^0 \rangle = A_T e^{-i\phi_T + i\delta_T} + A_P e^{-i\phi_P + i\delta_P} \equiv \bar{g} \quad (4)$$

the time-evolution of states with initially pure  $B^0$  and  $\bar{B}^0$  are found to be

$$\begin{aligned} \Gamma(B^0(t) \rightarrow \pi^+\pi^-) &\propto \frac{1}{1+a_\epsilon} \frac{(|g|^2+|h|^2)}{2} e^{-\Gamma t} [(1+a_{\epsilon\epsilon'}) \cosh(\Delta\Gamma t) \\ &+ (1+a_{\epsilon\epsilon'}) \sinh(\Delta\Gamma t) + (a_\epsilon + a_{\epsilon'}) \cos(\Delta mt) + a_{\epsilon+\epsilon'} \sin(\Delta mt)] \end{aligned} \quad (5)$$

$$\begin{aligned} \Gamma(\bar{B}^0(t) \rightarrow \pi^+\pi^-) &\propto \frac{1}{1-a_\epsilon} \frac{(|g|^2+|h|^2)}{2} e^{-\Gamma t} [(1+a_{\epsilon\epsilon'}) \cosh(\Delta\Gamma t) \\ &+ (1+a_{\epsilon\epsilon'}) \sinh(\Delta\Gamma t) - (a_\epsilon + a_{\epsilon'}) \cos(\Delta mt) - a_{\epsilon+\epsilon'} \sin(\Delta mt)] \end{aligned} \quad (6)$$

where  $a_\epsilon$ ,  $a_{\epsilon'}$ ,  $a_{\epsilon+\epsilon'}$  and  $a_{\epsilon\epsilon'}$  are rephase-invariant observables and defined as follows

$$\begin{aligned} a_\epsilon &= \frac{1 - |q/p|^2}{1 + |q/p|^2} = \frac{2\text{Re}\epsilon_B}{1 + |\epsilon_B|^2}, \quad a_{\epsilon'} = \frac{1 - |h/g|^2}{1 + |h/g|^2} = \frac{2\text{Re}\epsilon'_B}{1 + |\epsilon'_B|^2}; \\ a_{\epsilon+\epsilon'} &= \frac{-4\text{Im}(qh/pg)}{(1 + |q/p|^2)(1 + |h/g|^2)} = \frac{2\text{Im}\epsilon_B(1 - |\epsilon'_B|^2) + 2\text{Im}\epsilon'_B(1 - |\epsilon_B|^2)}{(1 + |\epsilon_B|^2)(1 + |\epsilon'_B|^2)} \\ a_{\epsilon\epsilon'} &= \frac{4\text{Re}(qh/pg)}{(1 + |q/p|^2)(1 + |h/g|^2)} - 1 = \frac{4\text{Im}\epsilon_B \text{Im}\epsilon'_B - 2(|\epsilon_B|^2 + |\epsilon'_B|^2)}{(1 + |\epsilon_B|^2)(1 + |\epsilon'_B|^2)} \end{aligned} \quad (7)$$

with  $\epsilon_B = (1 - q/p)/(1 + q/p)$  and  $\epsilon'_B = (1 - h/g)/(1 + h/g)$ . In the B system, since  $a_\epsilon \ll 1$ ,  $|\Delta\Gamma| \ll |\Delta m|$  and  $|\Delta\Gamma/\Gamma| \ll 1$ , the time-dependent asymmetry  $A_{CP}(t)$  can be simply written

$$A_{CP}(t) = \frac{\Gamma(B^0 \rightarrow f) - \Gamma(\bar{B}^0 \rightarrow f)}{\Gamma(B^0 \rightarrow f) + \Gamma(\bar{B}^0 \rightarrow f)} \simeq a_{\epsilon'} \cos(\Delta mt) + a_{\epsilon+\epsilon'} \sin(\Delta mt) \quad (8)$$

The CP-violating phase is related to the observables via [10]

$$\sin(2(\phi_M + \phi_A)) = \frac{a_{\epsilon+\epsilon'}}{\sqrt{(1 - a_\epsilon^2)(1 - a_{\epsilon'}^2)}} \quad (9)$$

where the phase  $\phi_M$  and  $\phi_A$  are defined by

$$\frac{q}{p} = \left|\frac{q}{p}\right| e^{-2i\phi_M}, \quad \frac{h}{g} = \left|\frac{h}{g}\right| e^{-2i\phi_A} \quad (10)$$

The tree amplitude is proportional to  $v_u$  whereas the penguin amplitude depends in general on  $v_u$  and  $v_c$ , where  $v_u = V_{ub}V_{ud}^*$ ,  $v_c = V_{cb}V_{cd}^*$  and  $v_t = V_{tb}V_{td}^*$ . It is well known that when the difference of the  $u$  and  $c$  contributions in the  $q\bar{q}$  intermediate states can be neglected, the penguin amplitude can be expressed in terms of  $v_t$  alone. A general analysis of these considerations has been carried through by Buras and Fleischer [11]. In the model to be considered in the next section this is only violated by the additional  $O(\alpha_s)$  and  $O(\alpha)$  corrections in the short distance coefficients [5]. In this approximation, for  $\bar{B}^0 \rightarrow \pi^+\pi^-$  decay, we have

$$\begin{aligned} h &= v_u \tilde{T} + v_t \tilde{P} \\ &= |v_u| T e^{-i\gamma} + |v_t| P e^{i\beta} \end{aligned} \quad (11)$$

Then we have

$$\begin{aligned} \phi_M &= \beta \\ \phi_T &= \gamma \\ \phi_P &= -\beta \end{aligned} \quad (12)$$

where  $\alpha$ ,  $\beta$  and  $\gamma$  are three angles of the unitarity triangle of the CKM matrix and  $\alpha = \pi - \beta - \gamma$ . Factoring the phase  $\phi_T$  of the tree contribution out we introduce the phase shift due to the penguins,  $\alpha_0$ , defined by:

$$\phi_A = \phi_T - \alpha_0 \quad (13)$$

As a result we can write

$$\frac{a_{\epsilon+\epsilon'}}{\sqrt{(1 - a_\epsilon^2)(1 - a_{\epsilon'}^2)}} = -\sin(2(\alpha + \alpha_0)) \simeq a_{\epsilon+\epsilon'} \quad (14)$$

where  $\alpha_0$  is given by

$$\begin{aligned} \tan 2\alpha_0 &= \frac{2(\frac{A_P}{A_T}) \sin \Delta\phi \cos \Delta\delta + (\frac{A_P}{A_T})^2 \sin(2\Delta\phi)}{1 + 2(\frac{A_P}{A_T}) \cos \Delta\phi \cos \Delta\delta + (\frac{A_P}{A_T})^2 \cos(2\Delta\phi)} \\ &= \frac{2(\frac{A_P}{A_T}) \sin \alpha \cos \delta - (\frac{A_P}{A_T})^2 \sin(2\alpha)}{1 - 2(\frac{A_P}{A_T}) \cos \alpha \cos \delta + (\frac{A_P}{A_T})^2 \cos(2\alpha)} \end{aligned} \quad (15)$$

with  $\Delta\phi \equiv \phi_T - \phi_P = \pi - \alpha$  the weak phase difference and  $\Delta\delta \equiv \delta_T - \delta_P \equiv \delta$  the strong phase difference between tree and penguin diagrams.

When the strong phase  $\delta$  is zero, this equation simplifies to:

$$\tan \alpha_0 = \frac{A_P}{A_T} \frac{\sin \alpha}{1 - \frac{A_P}{A_T} \cos \alpha} \quad (16)$$

(14) is a relation between the asymmetry  $a_{\epsilon+\epsilon'}$ , the unitarity triangle angle  $\alpha$ , and the penguin complication represented by  $\alpha_0$ . (15) shows how the angle  $\alpha_0$  depends on the size of the penguin, the strong phase  $\delta$  and the unitarity angle  $\alpha$  itself. To determine  $\alpha$  from the  $B^0/\bar{B}^0 \rightarrow \pi^+\pi^-$  time dependent asymmetry,  $\alpha_0$  must be calculated from a model or estimated from some other process. This is the subject of the next section.

### 3 Extraction of $\alpha$

The three angles  $\alpha$ ,  $\beta$  and  $\gamma$  are defined as

$$\alpha = \arg \left( -\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right), \quad \beta = \arg \left( -\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right), \quad \gamma = \arg \left( -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right), \quad (17)$$

They are related to the Wolfenstein parameters  $\rho$  and  $\eta$  as follows:

$$\tan \alpha = \frac{\eta}{\eta^2 - \rho(1 - \rho)}, \quad \tan \beta = \frac{\eta}{1 - \rho}, \quad \tan \gamma = \frac{\eta}{\rho} \quad (18)$$

From the present experimental data on  $|V_{ub}/V_{cb}|$ , one has [7]

$$\sqrt{\rho^2 + \eta^2} = 0.36 \pm 0.08 \equiv B \quad (19)$$

It is clear from (14) that to extract  $\alpha$  from experiment, one has to know  $\alpha_0$ . As shown in (15),  $\alpha_0$  depends on  $\alpha$  (or  $\eta$  and  $\rho$ ) as well as the ratio of the penguin amplitude  $A_P$  to the tree amplitude  $A_T$ . We can separate the CKM matrix elements and pure hadronic matrix elements in the amplitudes  $A_P$  and  $A_T$  in the following way,

$$A_T = |v_u| T, \quad A_P = |v_t| P \quad (20)$$

(20) is correct when we neglect the difference between the  $u$  and  $c$  quark contributions to the penguin amplitude, discussed further below. In this same approximation, the strong phase  $\delta$  is zero in our model; then in this limit (15) can be further simplified to:

$$\tan \alpha_0 = \frac{\sqrt{B^2 - \rho^2}}{B^2 - (B^2 - \rho)P/T} \left( \frac{P}{T} \right) \quad (21)$$

In general, calculating the strong phase  $\delta$  is difficult due to unknown nonperturbative effects. In [5] the strong phases derive from absorptive parts of the  $q\bar{q}$  intermediate states in the strong penguin contributions which are estimated perturbatively using recently developed next to leading log formalism following the method pioneered by [12]. In [5] it is found that

$$\delta_P \simeq 9.5^\circ, \quad \delta \simeq -9.5^\circ, \quad \delta_T = 0 \quad (22)$$

For a general consideration, we take  $\delta$  as a free parameter. The ratio  $(P/T)$  is purely determined by the hadronic matrix elements. In the operator product expansion approach, the hadronic matrix elements are products of short-distance parts, i.e. Wilson coefficients, evaluated by perturbative QCD, and a badly known long-distance part. In a factorization approximation the long distance hadronic matrix elements are themselves products of current form factors and decay (coupling) constants. For the  $B^0 \rightarrow \pi^+\pi^-$  decay, it is easy to see that the ratio  $(P/T)$  is almost independent of the uncertainties in the long-distance modeling because differences in the approaches cancel in the ratio. Therefore,  $\alpha_0$  can be well determined in a rather model independent way and given by coefficients  $a_i$  which have been defined in [5] in terms of the effective Wilson coefficients  $c_i^{eff}$ :

$$a_i = c_i^{eff} + \frac{1}{N} c_j^{eff} \quad (23)$$

where (i,j) is any of the pairs (1,2), (3,4), (4,5), (6,7), (7,8) and (9,10) and N is the number of colors. (See [5] for further details.) The second term in (23) arises from the Fierz rearrangements in connection with the factorization contributions. In [5] two models were considered,  $N = \infty$  and  $N = 2$ , to account for possible non-factorizable contributions. Then as one can see from Tab. 1a,b of [5] the ratio  $|P/T|$  becomes:

$$\left| \frac{P}{T} \right| = \frac{|a_4 + a_{10} + (a_6 + a_8)R[\pi^-, \pi^+]|}{|a_2|} \quad (24)$$

As we see from (24) this ratio does not depend on the current form factors and decay constants. It depends only on the factorization hypothesis and on the effective short distance coefficients. Furthermore it is found that the ratio is not sensitive to the effective color number N in (23). The reason for the simple structure of  $|P/T|$  as given by (24) is that for  $\pi^+\pi^-$  states there is only one way to factorize the transition matrix element.

The tree and penguin amplitude for  $B^0 \rightarrow \pi^+\pi^-$  were evaluated in [5] for various hypotheses concerning the  $O(\alpha_s)$  corrections in the  $c_i^{eff}$  (which include the absorptive parts) and the influence of the electroweak penguins. From these results we can calculate the amplitudes T and P with the CKM phases factored out. Since the current matrix elements cancel in the ratio this is equivalent to evaluating P/T in terms of the coefficients  $a_i$ . The results are displayed in Tab. 1 for  $N=2$  and  $N=\infty$ . The notation  $P_y^x$  refers to penguin amplitudes arising from  $x = st$  (strong) or  $x = ew$  (electroweak) penguins, and  $y = u$  or  $y = c$  parts of the weak Hamiltonian with and without  $O(\alpha_s)$  terms in  $c_i^{eff}$ , where the absorptive parts are contained in the  $O(\alpha_s)$  corrections. In the following we shall use these results in order to calculate  $\alpha_0$  for various assumptions concerning  $O(\alpha_s)$  terms in the  $c_i^{eff}$  or electroweak penguin effects. The simplest case is to use the tree and strong penguin amplitudes with  $O(\alpha_s)$  corrections neglected, i.e. the penguin amplitudes of column 3 and 4 in Tab. 1 which results in  $\delta = 0$ . The relation between these amplitudes and the penguin introduced above is  $P = -\frac{1}{2}(P_u^{st''} + P_c^{st''})$ . For this case  $\frac{P}{T}$  in (24) is 0.05 for both  $N=2$  and  $N = \infty$ . For fixed  $B$  (21) determines  $\alpha_0$  as a function of  $\alpha$  using (18).

In Fig. 1  $\alpha_0$  is plotted as a function of  $\alpha$  with  $B = (\rho^2 + \eta^2)^{\frac{1}{2}} = 0.28, 0.36, 0.44$ , respectively, for  $\delta = 0$ . From Fig. 1 it is apparent that the penguin shift decreases with increasing  $B$ . The maximum of  $\alpha_0$  as a function of  $\alpha$  occurs near  $\rho = 0$ . In the plot  $\rho$  varies with  $\alpha$  starting from  $\rho < 0$  to  $\rho > 0$  with increasing  $\alpha$ . Fig. 1 contains our main result. As one can see the penguin shift,  $\alpha_0$ , is small compared to  $\alpha$ .  $\frac{\alpha_0}{\alpha}$  is largest at small  $\alpha$  and decreases monotonically with increasing  $\alpha$  to zero.

To see the influence of a strong phase  $\delta$  we have repeated the calculation of  $\alpha$  now using (15)

where we still use the same ratio of  $P/T$  from column 3 and 4 in Tab. 1. In Fig. 2. the angle  $\alpha_0$  is plotted as a function of  $\alpha$  for  $B=0.36$  and four values of the strong phase,  $\delta = 0^\circ, 10^\circ, 40^\circ, 90^\circ$ . The maximal distortion occurs for  $\delta = 0^\circ$  as is evident already from (15). The behavior for  $90^\circ < \delta < 180^\circ$  is approximately a reflection  $\alpha_0(\delta) = -\alpha_0(\pi - \delta)$ .  $\alpha_0(\delta)$  is an even function of  $\delta$ .

The relation between  $\alpha$  and the measured angle  $\alpha_M$  is  $\alpha \equiv \alpha_M - \alpha_0$ , which can be read off from Fig. 3. This plot is for  $\delta = 10^\circ$  and  $B= 0.28, 0.36, 0.44$ . As we can see,  $\alpha$  as a function of  $\alpha_M = \alpha + \alpha_0$  which comes from the measurement of  $a_{\epsilon+\epsilon'}$  is rather independent of  $B$  and differs only slightly from the straight line of slope unity it would be, if there were no penguin contributions.

The results for  $\alpha_0$  presented so far are for Wilson coefficients in which  $O(\alpha_s)$  and electroweak contributions have been neglected. The  $O(\alpha_s)$  terms yield deviations from (20) and additional absorptive contributions which generate the phase in (22). In Tab. 1 the penguin terms with the  $O(\alpha_s)$  terms included are given in the fifth and sixth columns as  $P_u^{st}$  and  $P_c^{st}$ . As we can see the  $O(\alpha_s)$  terms change the penguin terms up to 24% and produce the deviation

$$\frac{P_u^{st} - P_c^{st}}{P_u^{st} + P_c^{st}} = 0.13$$

in the real parts. In addition there is an imaginary part of the same order as the real part (these numbers are for the  $N=2$  case). Of course the resulting shift in  $\alpha_0$  depends on  $\alpha$  or equivalently on the value of  $\rho$ . Instead of calculating  $\alpha_0$  as a function of  $\alpha$  (or  $\rho$ ) we quote only results where  $\alpha$  nearly had its maximum, i.e.,  $\rho = 0$ , at  $\alpha = 70^\circ$ . By calculating  $a'_\epsilon$  and  $a_{\epsilon+\epsilon'}$  directly we can use (14) to extract  $\alpha_0$  for this value of  $\alpha$ . The major effect is due to the strong penguin amplitude itself, without  $\alpha_s$  or electroweak corrections, which shifts  $\alpha$  by  $\alpha_0 = 8.0^\circ$ . The  $\alpha_s$  and electroweak corrections shift  $\alpha$  by an additional  $\Delta\alpha = 1.9^\circ$  and  $0.6^\circ$  respectively, to a total shift of  $\alpha_0 = 10.5^\circ$ .

The values of the strong phase may be extracted from the direct CP-violating parameter  $a_{\epsilon'}$ . Hence in principle one can determine  $\delta$  without recourse to a model. On general grounds, however, one would expect  $\delta$  to be small.

It is clear that the shift  $\alpha_0$  due to the penguin effects is small. For all  $\alpha$  the relative shift  $\alpha_0/\alpha$  is less than 30% and decreases from this value with increasing  $\alpha$ . This result depends on our model of the penguin amplitudes, in particular on  $P/T$ . In principle one need not rely on the model but rather obtain  $|P|$  from pure penguin or penguin dominated processes, i.e.,  $\bar{K}^0\pi^-$  (pure penguin) and  $K^+\pi^-$  (penguin dominated). Unfortunately there are only experimental upper bounds on the branching ratios of these decays. In reference [5] we have calculated the branching ratios of these decays for the special choice  $\rho = -0.12, \eta = 0.34$ . Since  $\bar{K}^0\pi^-$  depends only on  $|V_{ts}|$  which is well known, we can obtain upper limits on  $|P|$ . Taking for example the  $N=2$  case, when we compare with the experimental limit [13],  $BR(B^- \rightarrow \bar{K}^0\pi^-) < 4.8 \times 10^{-5}$ , we find that  $|P_{exp}/P| < 2.2$ . Keeping  $T$  fixed, such a large value of the penguin amplitude given by the upper limit of  $|P_{exp}|$  would increase  $\alpha_0$  from  $8^\circ$  to  $17^\circ$  for the maximal shift at  $\rho = 0$ . Here we make the assumption that the penguin amplitudes of  $K\pi$  and  $\pi\pi$  final states are related, i.e. a larger  $P$  in  $K\pi$  would mean a larger  $P$  in  $\pi\pi$ . This can be justified with  $SU(3)$  symmetry arguments.  $SU(3)$  symmetry breaking effects are indeed moderate in our model calculations.

The penguin dominated decay channel  $\bar{B}^0 \rightarrow K^+\pi^-$  gives us a much better limit as advocated by Silva and Wolfenstein [14]. The experimental limit on this branching ratio is  $1.7 \times 10^{-5}$ ; from our previous work we know that this decay is dominated by the penguin amplitude (since the tree amplitude is Cabibbo suppressed) in the ratio 4:1 in the ampli-

tude. This gives about  $|P_{exp}/P| < 1.4$  leading to an even smaller shift of  $\alpha_0$  to  $\simeq 11^\circ$  compared to the  $8^\circ$  calculated above. Other experimental limits on the branching ratios relevant for comparing with our model are  $BR(\pi^-\pi^0) < 1.6 \times 10^{-5}$ ,  $BR(\pi^+\pi^-) < 2.0 \times 10^{-5}$  and  $BR(\pi^+\pi^- + K^+\pi^-) = (1.8 \pm 0.6) \times 10^{-5}$  [13]. Our model obeys these constraints; in particular we obtained  $BR(\pi^+\pi^- + K^+\pi^-) = 2.15 \times 10^{-5}$  for  $\rho = 0, \eta = B$  which agrees perfectly with the measured value. This shows that the tree and penguins can not be too far from the values in our model.

In conclusion, we find that the penguin distortion in the determination of  $\alpha$  from the  $\pi^+\pi^-$  asymmetry is not a real obstacle provided  $\alpha$  is not too small. Even if the penguin amplitude is taken from the upper limit of the pure penguin dominated process the penguin distortion on  $\alpha$  remains below  $\sim 25\%$  at large  $\alpha$ . Improved experimental work on exclusive charmless hadronic B decays will even more sharply constrain the size of penguin amplitudes and in turn limit the shift  $\alpha_0$ .

## References

- [1] For reviews see, Y. Nir, H. R. Quinn, in “B Decays”, edited by S. Stone, World Scientific, Singapore, 1992, p. 362; I. Dunietz, *ibid*, p. 393.
- [2] M. Gronau and D. London, Phys. Rev. Lett. 65 (1990) 3381.
- [3] N. G. Deshpande and X.G. He, Phys. Rev. Lett. 74 (1995) 26, 4099(E).
- [4] M. Gronau et al., TECHNION-PH-95-11, hep-ph/9504327.
- [5] G. Kramer and W. F. Palmer, DESY 95-131, 1995, Phys. Rev. D., in press.
- [6] F. DeJongh and P. Sphicas, Fermilab report FERMILAB-PUB-95/179, July 1995.
- [7] A. Ali and D. London, Z. Phys. C65 (1995) 431.
- [8] G. Anderson, S. Dimopoulos, L.J. Hall, S. Raby and G.D. Starkman, Phys. Rev. D49 (1994) 3660; K.C. Chou and Y.L. WU, to appear in Scientia Sinica, the first issue, 1996, hep-ph/9508402.
- [9] K.C. Chou and Y.L. Wu, CAS-HEP-T-95-11/003, OHSTPY-HEP-T-95-023, hep-ph/9511327, 1995.
- [10] W.F. Palmer and Y.L. Wu, Phys. Lett. B350 (1995) 245.
- [11] A. Buras and R. Fleischer, Phys. Lett. B341 (1995) 379.
- [12] M. Bander, D. Silverman, A. Soni, Phys. Rev. Lett 43 (1979) 242.
- [13] D. M. Asner et al., CLEO Collaboration, report CLNS 95/1338, CLEO 95-8 (1995).
- [14] J. Silva and L. Wolfenstein, Phys. Rev. D 49 (1994) R1151.



## Table Caption

**Tab. 1:** Reduced amplitudes for  $\bar{B}^0 \rightarrow \pi^+\pi^-$ . CKM phases have been removed. These amplitudes are based on a next to leading log corrected weak Hamiltonian with  $\alpha_s$  and  $\alpha$  corrections, assuming factorization of the hadronic matrix elements. Numbers in the parenthesis are the real and imaginary part of the amplitude. For further details, see reference [5].

Tab. 1

Reduced Amplitudes $B^0 \rightarrow \pi^+\pi^-$ Tree, Strong and EW Penguins $T, P^{st}, P^{ew}$ with ( ) or without (") $\alpha_s$ corrected NLL QCD Coefficients							
N	T	$P_u^{st''}$	$P_c^{st''}$	$P_u^{st}$	$P_c^{st}$	$P_u^{ew}$	$P_c^{ew}$
$\infty$	2.84	(-0.141,0)	(-0.141,0)	(-0.134,-0.0548)	(-0.175,-0.0291)	(0.0052,0)	(0.0052,0)
2	2.44	(-0.115,0)	(-0.115,0)	(-0.109,-0.0457)	(-0.143,-0.0243)	(0.0052,0)	(0.0052,0)

## Figure Caption

**Fig. 1.**  $\alpha_0$  as a function of  $\alpha$  with  $\delta = 0^\circ$ . The curves correspond to  $B = 0.28$  (dotted), 0.36 (solid), and 0.44 (dashed). All angles are in degrees.

**Fig. 2.**  $\alpha_0$  as a function of  $\alpha$  with  $B = 0.36$ . The curves correspond to the strong phase  $\delta = 0^\circ$  (dashed),  $10^\circ$  (solid),  $40^\circ$  (dotted), and  $90^\circ$  (dot-dashed). All angles are in degrees.

**Fig. 3.**  $\alpha = \alpha_M - \alpha_0$  as a function of the measured  $\alpha_M$  with  $\delta = 10^\circ$ . The curves correspond to  $B = 0.28$  (dotted), 0.36 (solid), and 0.44 (dashed). All angles are in degrees.

This figure "fig1-1.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9512341v1>

This figure "fig1-2.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9512341v1>

This figure "fig1-3.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9512341v1>